section 3
sheet 3

③ $\vec{F}(x,y,z) = yz\vec{i} + xz\vec{j} + xy\vec{k}$

$\vec{r} = t\vec{i} + t^2\vec{j} + t^3\vec{k} \quad ; 0 \le t \le 1$

$dr = \vec{i} + 2t\vec{j} + 3t^2\vec{k} \qquad \vec{F}(r(t)) = t^5 i + t^4\vec{j} + t^3\vec{k}$

$$W = \oint_C \vec{F}\cdot d\vec{r} = \oint_C \left(\vec{F}(r(t)) \cdot \frac{d\vec{r}}{dt}\right) dt$$

$$= \int_0^1 (t^5\vec{i} + t^4\vec{j} + t^3\vec{k})(\vec{i} + 2tj + 3t^2k)\,dt$$

$$= \int_0^1 (t^5 + 2t^5 + 3t^5)\,dt = \int_0^1 6t^5\,dt$$

$$= t^6$$

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(4) $\oint_C (3x - 4y)\,dx + (4x + 2y)\,dy$

$C = \frac{x^2}{16} + \frac{y^2}{9} = 1 \qquad 0 \le t \le 2\pi$

parametric equation

$x = 4\cos t \qquad y = 3\sin t$

$dx = -4\sin t\,dt \qquad dy = 3\cos t\,dt$

$$\int_0^{2\pi} (12\cos t - 12\sin t) - 4\sin t\,dt + (16\cos t + 6\sin t)\,3\cos t\,dt$$

$$\int_0^{2\pi} \cancel{-48\cos t\sin t}\,dt + 48\sin^2 t\,dt + 48\cos^2 t\,dt \cancel{+ 18\sin t\cos t\,dt}$$

$$\cancel{\int_0^{2\pi} 48(1)\,dt} = \int_0^{2\pi} -30\cos t\sin t\,dt + 48$$

$$= -\frac{30\sin^2 t}{2} + 48t \Big|_0^{2\pi} = 48 \times 2\pi = 96\pi$$

$\boxed{5}$ $I = \int_{(1,2)}^{(3,4)} (6xy^2 - y^2)\,dx + (6x^2y - 3xy^2)\,dy$

Sol.

Let $(6xy^2 - y^3) = M$

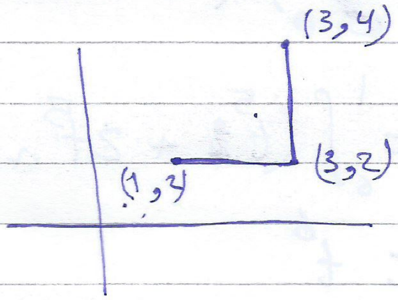
$\frac{\partial M}{\partial y} = (12xy - 3y^2)$

Let $(6x^2y - 3xy^2) = N$

$\frac{\partial N}{\partial x} = (2xy - 3y^2)$

or

$\nabla \times F = 0$



on C_1: $y = 2$, $dy = 0$, $1 \le x \le 3$

$I_1 = \int_1^3 24x - 4\,dx = 12x^2 - 4x \Big|_1^3$

$= 108 - 12 - 12 - 4 = 80$

on C_2: $x = 3$, $dx = 0$, $2 \le y \le 4$

$I_2 = \int_2^4 18y - 9y^2\,dy = 9y^2 - 3y^3 \Big|_2^4$

$96 - 12 = 156$

$I = I_1 + I_2 = 236$

(8) $I = \int_{(1,0)}^{(2,1)} (2xy - y^4 + 3)\,dx + (x^2 - 4xy^3)\,dy$

$$\frac{\partial M}{\partial y} = 2x - 4y^3$$

$$\frac{\partial N}{\partial x} = 2x - 4y^3$$

$$\vec{F} = \vec{\nabla}\varphi$$

$$\varphi_x = 2xy - y^4 + 3 \longrightarrow \varphi = x^2y - xy^4 + 3x + \cancel{A(y)}^{\text{zero}}$$

$$\varphi_y = x^2 - 4xy^3 \longrightarrow \varphi = x^2y + y^4 + \boxed{A_2(x)} \hookrightarrow = 3x$$

$$\varphi = x^2y - xy^4 + 3x$$

$$I = \varphi(2,1) - \varphi(1,0) = (4 - 2 + 6) - (0 - 0 + 3)$$

$$= +5$$